IB Math HL Y1
Binomial Theorem
Name ________________________________ Date __________

Part 1 – Basics

The **Binomial Theorem** is used for expanding expressions of the form $(a+b)^n$. It states

$$(a+b)^n = \sum_{r=0}^{n} {}_nC_r a^{n-r} b^r = {}_nC_0 a^n + {}_nC_1 a^{n-1} b + {}_nC_2 a^{n-2} b^2 + \ldots {}_nC_n b^n$$

1. Besides combinations, how else can you find the coefficients used in the Binomial Theorem? Use the space below to show how this works.

2. Expand $(x-5)^3$ using the Binomial Theorem.

3. Find the coefficient of x^4 in $(2x+3)^7$.

4. Find the coefficient of $\frac{1}{x}$ in the expansion of $\left(x-\frac{2}{x}\right)^5$.

Part 2 – Practice

The ***constant term*** of an expansion is a term where all variables are raised to the 0 power.

5. Find the constant term of

   (a) $\left(3x^2 - \frac{2}{x}\right)^6$

   (b) $(2+x)\left(2x+\frac{1}{x}\right)^5$

6. The coefficient of x in the expansion of $\left(x+\frac{1}{ax^2}\right)^7$ is $\frac{7}{3}$. Find the possible values of a.

7. Expand $(2+x)^5$ and hence find 1.9^5 (without a calculator).

Part 3 – Extension – Extended Binomial Theorem

8. Write in terms of n.

   (a) $\binom{n}{1}$ (b) $\binom{n}{2}$ (c) $\binom{n}{3}$

Amazingly, if we use the above formulas (and their generalizations), it can be shown that the Binomial Theorem holds true for values of n which are not positive integers! In this case, the expansion of $(a+b)^n$ is infinite. Thus we either have to concern ourselves with deciding when the infinite series converges, or we limit ourselves to a finite number of terms.

7. Use the Extended Binomial Theorem to write the indicated number of terms.

   (a) $(1+x)^{1/2}$ 3 terms

   (b) $(1-x)^{-1}$ 4 terms

8. Suppose we could show that the pattern from 7(b) continues for other terms – it does!

   (a) Write out the result as an infinite sum.

   $(1-x)^{-1} =$

   (b) What result does this remind you of? Explain how this tells us when the infinite series converges.

9. In science, the approximation $(1+x)^n \approx 1+nx$ is often used for small values of x. Show where this approximation formula comes from.